

SECOND PROBLEM SET

Math 5615H: Honors Analysis

Due W 20 September, 2017.
10 points each; total 50 points.

1. Let F be a field. Show that there exist **at most two** solutions of $x^2 = 1$.
True/false: Is it possible that there exists only one solution?
2. Given any two **rational** numbers p and q , with $p < q$, show that there is an **irrational** number x with $p < x < q$.
3. **(a)**. Represent the polynomial $z^4 + 4$ in the form

$$z^4 + 4 = (z - c_1)(z - c_2)(z - c_3)(z - c_4)$$

for some complex constants c_1, c_2, c_3, c_4 .

(b). Write $z^4 + 4$ as the product of two quadratic polynomials with **real** coefficients.

4. Problem #6 on p. 22.
5. Problem #10 on p. 22.