Math 2263
Spring 2016
Midterm 2
March 24, 2016
Time Limit: 50 minutes

Name (Print): ______________________
Student ID: ______________________
Section Number: __________
Teaching Assistant: ______________________
Signature: ______________________

This exam contains 8 pages (including this cover page) and 7 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated. **Calculators may be used, but are not necessary.** Please turn off cell phones. **Crib sheet:** You are allowed to bring one single - sided 8.5 inch $\times$ 11 inch sheet of notes to the exam.

Do not give numerical approximations to quantities such as $\sin 5$, π, or $\sqrt{2}$. However, you should simplify $\cos\frac{\pi}{2} = 0$, $e^0 = 1$, and so on.

The following rules apply:

- **Show your work**, in a reasonably neat and coherent way, in the space provided. **All answers must be justified by valid mathematical reasoning.** To receive full credit on a problem, you must show enough work so that your solution can be followed by someone without a calculator.
- **Mysterious or unsupported answers will not receive full credit**. Your work should be mathematically correct and carefully and legibly written.
- **A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit**; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

1	15 pts	
2	15 pts	
3	15 pts	
4	25 pts	
5	25 pts	
6	15 pts	
7	20 pts	
8	20 pts	
TOTAL	150 pts	

1. (16 points) Let D be the region $0 \leq x \leq 4$, $0 \leq y \leq \sqrt{x}$ in the (x, y)-plane. Find the double integral
$$\iint_D x^2 y \, dA.$$

2. (18 points) Let E be the triangular solid

$$E = \{(x, y, z) : x \geq 0,\ y \geq 0,\ x + y \leq 1,\ 0 \leq z \leq 1\}.$$

Find the triple integral

$$\iiint_E xyz\, dV.$$

3. (16 points) Let R be the rectangle $-3 \leq x \leq 4$, $0 \leq y \leq 3$ in the (x, y)-plane. If a continuous function $f(x, y)$ satisfies

$$-|x|y^2 \leq f(x, y) \leq 2,$$

for all $(x, y) \in R$, what does this tell you about the value of $\iint_R f(x, y)\, dA$?

4. (28 points) Suppose $x = X$ and $y = Y$ are random variables with joint density function

$$f(x, y) = \alpha(x^2 + y^2) \quad \text{if} \quad x^2 + y^2 \leq 1,$$

and

$$f(x, y) \equiv 0 \quad \text{if} \quad x^2 + y^2 > 1.$$

(a) (16 points) What does the constant α need to be?

(b) (12 points) Find the "median" radius R, so that the probability that $X^2 + Y^2 \leq R^2$ is 50%.

5. (28 points) A plate (or lamina) is in the shape of the triangle D: $0 \leq x \leq 2$, $0 \leq y \leq 1 - \frac{x}{2}$, with corners $(0, 0)$, $(0, 1)$ and $(2, 0)$. The plate has mass density at the point (x, y) equal to $\rho(x, y) = xy$ per unit area.

(a) (12 points) Find the **total mass** m of the plate.

(b) (16 points) Find the **center of mass** $(\overline{x}, \overline{y})$ of the plate.

6. (22 points) Use the method of **Lagrange multipliers** to find the **maximum** and **minimum** values of

$$f(x, y) = xy$$

among points (x, y) which lie on the ellipse

$$g(x, y) = x^2 + xy + y^2 = 3.$$

7. (22 points) Compute the surface area $A(S)$ of the parabolic hyperboloid $S = \{(x, y, z) : x^2 + y^2 \leq 1, z = y^2 - x^2\}$. (*Hint:* polar coordinates.)