Math 2263	**Name (Print):** ______________
Fall 2008	**Student ID:** ______________
Midterm 1	**Section Number:** _______
October 2, 2008	**Teaching Assistant:** ______________
Time Limit: 50 minutes	**Signature:** ______________

This exam contains 8 pages (including this cover page) and 7 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated. Calculators may be used. Please turn off cell phones.
Do not give numerical approximations to quantities such as $\sin 5$, π, or $\sqrt{2}$. However, you should simplify $\cos \frac{\pi}{2} = 0$, $e^0 = 1$, and so on.

The following rules apply:

- **Show your work**, in a reasonably neat and coherent way, in the space provided. **All answers must be justified by valid mathematical reasoning.** To receive full credit on a problem, you must show enough work so that your solution can be followed by someone without a calculator.

- **Mysterious or unsupported answers will not receive full credit**. Your work should be mathematically correct and carefully and legibly written.

- **A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit**; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

1	15 pts	
2	15 pts	
3	15 pts	
4	15 pts	
5	10 pts	
6	15 pts	
7	15 pts	
TOTAL	100 pts	

1. (15 points) The lines given parametrically by

$$\langle x, y, z\rangle = \langle 5 - t, 3 + 2t, 1 + 2t\rangle, \quad -\infty < t < \infty$$

and

$$\langle x, y, z\rangle = \langle 5 + 2s, 3 + 2s, 1 - s\rangle, \quad -\infty < s < \infty$$

intersect at the point $\langle x, y, z\rangle = \langle 5, 3, 1\rangle$. Find an equation for the **plane** which contains both lines.

2. (15 points) Find an equation for the **elliptical cylinder** in (x, y, z)-space containing infinitely many lines parallel to the z-axis, and containing the slanted circle $z = y$, $x^2 + y^2 + z^2 = 4$.

3. (15 points) Evaluate the limit

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - xy + y^2}{x^2 + xy + y^2},$$

or state that it does not exist, giving reasons.

4. (15 points) For the function

$$f(x, y) = e^{3y} \cos 2x,$$

find the **second partial derivatives**

$$f_{xx} = \frac{\partial^2 f}{\partial x^2}, \quad f_{xy} = \frac{\partial^2 f}{\partial y \partial x} \quad \text{and} \quad f_{yy} = \frac{\partial^2 f}{\partial y^2}.$$

5. (10 points) Suppose $z = f(x, y)$ is a function with partial derivatives $f_x(3, 4) = 3$ and $f_y(3, 4) = -2$. If x and y are both functions of t: $x = 4 - t^2$ and $y = 3t + t^2$, find

$$\frac{dz}{dt} = \frac{d}{dt} f\big(x(t), y(t)\big)$$

at $t = 1$.

6. (15 points) The point $\langle x, y, z\rangle = \langle -2, 1, 0\rangle$ lies on the surface S:

$$x^2 - y^2 + xz + xy - 4z^2 = 1.$$

Find the equation of the **tangent plane** to the surface S at $\langle -2, 1, 0\rangle$, in the form $ax + by + cz = d$.

7. **(15 points) (a)**Find the **gradient** of the function $f(x, y, z) = (x + z^2)\sin(xy)$ at the point $\langle x, y, z\rangle = \langle 1, \frac{\pi}{2}, 2\rangle$.

(15 points) (b) Find the **directional derivative** of f at the point $\langle 1, \frac{\pi}{2}, 2\rangle$ in the direction of the unit vector

$$\vec{u} = \frac{1}{3}\left(2\vec{i} - \vec{j} - 2\vec{k}\right).$$