

Let a be a function in $L^\infty(\Gamma)$, where $\Gamma = \Gamma(t)$ is a Carleson curve, that is a curve such that $\sup_{t \in \Gamma} \sup_{\epsilon > 0} |\Gamma(t, \epsilon)| \epsilon$ is finite, where the symbol $|\Gamma(t, \epsilon)|$ denotes the measure of the part $\Gamma(t, \epsilon)$ of the curve Γ in the disc $|\tau - t| < \epsilon$. Let $w(\tau)$ denote a Muckenhaupt weight defined on Γ ; the class of Muckenhaupt weights is denoted by $A_p(\Gamma)$. The complex function f is said to be in $L^p(\Gamma, w)$ if $(\int_\Gamma |f(\tau)|^p w(\tau) d\tau)^{1/p} = \|f\|_{p,w}$ is finite. For f in the Hardy space $H^p(\Gamma, w)$ the Toeplitz operator $T(a)$ is the projection of the function of a onto the Hardy space $H^p(\Gamma, w)$. The essential spectrum of an operator T is the set of complex numbers λ such that $T - \lambda I$ is not Fredholm; I is the identity operator. The essential spectrum of the Toeplitz operator $T(a)$ is described in terms of indicator functions. The function $V^{(0)}_t \psi(\mathcal{E})$ is defined by the equations $V^{(0)}_t \psi(\mathcal{E}) = \lim_{R \rightarrow \infty} \exp \exp \int_{\Gamma(t, \mathcal{E}R)} \log \psi(r) |dr| / |\Gamma(t, \mathcal{E}R)| / \exp \int_{\Gamma(t, R)} \log \psi(r) |dr| / |\Gamma(\mathcal{E}R)|$ for $0 < \mathcal{E} < 1$, $V^{(0)}_t \psi(\mathcal{E}) = \lim_{R \rightarrow 0} \sup \exp \int_{\Gamma(t, R)} \log \psi(r) |dr| / |\Gamma(t, r)| / \exp(\int_{\Gamma(t\mathcal{E}^{-1}R)} \log \psi(r) |dr| / |\Gamma(t, \mathcal{E}^{-1}R)|)$ for $\mathcal{E} > 1$. The indicator function $\alpha_t(x)$ is defined by the formula $\alpha_t(x) = \limsup_{\mathcal{E} \rightarrow 0} V^{(0)}_t \exp(-x \arg(\tau - t)w(\mathcal{E}) / \log \mathcal{E})$ and $\beta_t(x)$ is the limit superior of the same function as $\mathcal{E}$ tends to infinity. It is shown that if a is piecewise continuous on Γ , then the essential spectrum of $T(a)$ consists of the essential range $\mathcal{R}(a)$ and the leaves $\cup_{t \in \Lambda_a} \mathcal{L}(a(t-0), a(t+0), p, \alpha_t, \beta_t)$ where Λ_a denotes the set of points of discontinuity of the function a on Γ and, for points, z, w , the symbol $\mathcal{L}(z, w, p, \alpha_t, \beta_t)$ denotes the leaf about z and w determined by p, α_t, β_t that is the set of points $\{w(\exp 2\pi\gamma - z) / (\exp 2\pi\gamma - 1)\}$ as γ ranges over the strip $1/p + \alpha_t(x) \leq y \leq 1/p + \beta_t(x)$ $\gamma = x + iy$. If λ is not in the essential spectrum of $T(a)$, then the index of $T(a) - \lambda I$ is the winding number of the curve $\mathcal{R}(a) \cup_{t \in \Gamma_a} \mathcal{L}(a(t-0), a(t+0), p, (\alpha_t + \beta_t)/2, (\alpha_t + \beta_t)/2)$ about the point λ . The indicator set N_t of Γ, p, w at the point t is the set of complex numbers γ such that $|\tau - t|^x \exp(-y \arg(\tau - t))w$ is a Muckenhaupt weight, where $\gamma = x + iy$. It is shown that N_t is the set of complex numbers γ such that $-1/p < x + \alpha_t(y) \leq x + \beta_t(y) < p/(p-1)$. A plane set N is said to be narrow if it is contained in a set $\pi = S_1 \cap S_2$ where each set S_i is a strip of width at most 1 and $\inf_{x+iy \in \pi} y = \inf_{x+iy \in N} y, \sup_{x+iy \in \pi} y = \sup_{x+iy \in N} y$. It is shown that open convex narrow sets containing the origin are precisely the sets which coincide the indicator set N_t for some Carleson curve Γ and some weight in $A_p(\Gamma)$ for some $t \in \Gamma$.