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Let X be a topological space and let (Y, ρ) be a metric space with metric ρ. A net $\mathcal{F} = \{f_i\}_{i=\mathcal{I}}$ from X to Y is said to be α-convergent to the function f from Y to y if for all $\epsilon > 0$ there exists an $i_0 \in \mathcal{J}$ and κ_o in K such that for all i in $\mathcal{J}$, $i > t_0$ and κ in $K \cap \kappa > x_0$, $\rho(f_i(x_\kappa),\ f(x)) < \epsilon$ for all x in X.

A net of functions $\mathcal{F} = \{f_i\}_{i=2}$ from x to (Y_p) is said to be exhaustive at a point x_0 of X if for all $\epsilon > 0$ there an open set V containing the point x_0 and an i_0 in $\mathcal{J}$ such that for all x in V and $i > i_0$ $\rho((x_0), f_i(x) < \epsilon$. The net $\mathcal{F}$ is said to be exhaustive if it is exhaustive for all points of X. It is shown that if $(X, d), (Y, \rho)$ are metric spaces and $\{f_n\}$ is a function from X to Y, then f_n converges to the function of if and only if $\{f_n\}$ converges pointwise to f and $\{f_n\}$ is exhaustive. If (X, d), (Y, ρ) are metric spaces $\mathcal{F}$ is a family of functions from X to Y it is proved that $\mathcal{F}$ is equicontinuous at a point x in X if and only if $\mathcal{F}$ is exhaustive at x and each function f in $\mathcal{F}$ is continuous at x. The following generalized Ascoli theorem is proved: If X is a compact metric space and $\mathcal{F}$ is an infinite closed bounded and exhaustive subset of $Bd(X)$, the space of bounded real valued functions on X with the supremum norm, then $\mathcal{F}$ is compact, in the topology of $Bd(X)$.