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Review text:

The concept of slow oscillation and moderate divergence are extended to linear topological space. If $x = \{x_n\}$ is a sequence of elements in a linear topological space B , and $S_n(x) = \sum_{k=1}^n x_k$ are the partial sums of the series $\sum x_k$, then for each linear functional Φ on B the sequence $\{S_n(x)\}$ is said to be Φ -slowly oscillating if $\lim |\Phi(S_N(x)) - \Phi(S_M(x))| = 0$ as M, N tend to infinity, $N > M$, N/M tends to one. If the sequence $\{S_n(x)\}$ is Φ -slowly oscillating for each functional Φ in the dual B^* of B , then the sequence $\{S_n(x)\}$ is said to be weakly-slowly oscillating. If B is normed and $\lim \|S_N(x) - S_M(x)\| = 0$ as M, N tend to infinity $N > M$, N/M tends to one, then $\{S_n(x)\}$ is said to be slowly oscillating in norm. If B is normed then the sequence $\{S_n(x)\}$ is said to be moderately divergent if $\|S_n(x)\| = o(n^{s-1})$ as n tends to infinity and $\sum_{n=0}^{\infty} \|S_n(x)\|/n^s < \infty$ for $s > 1$. It is shown that if $\{S_n(x)\}$ is weakly slowly oscillating in B , then the series $\sum \Phi(x_n) \exp i n t / n$ is the Fourier series of a function in $L^2(0, 2\pi)$ for $r \geq 2$. if $\{S_n(x)\}$ is moderately divergent then for each functional Φ in B^* there is an absolutely convergent Fourier representing a function α_Φ whose derivative α'_Φ is in L^r for $r \geq 2$; moreover the function α_Φ satisfies the relation $\sum_{n=1}^{\infty} \Phi(x_n/n) \exp i n t = \alpha_\Phi(t) - i (1 - \exp i t)$. For each functional Φ in B^* the Fourier series $\sum_{n=1}^{\infty} \Phi(x_n) \exp i n t$ is absolutely convergent if and only if the quantities $\|\sum_{k=1}^n (\exp i a r g \Phi(x_k)) x_k\|$ are bounded.