

Reviewer: George U. Brauer

Reviewer number: 003307

Address:

Math Depart.
University of Minnesota
Minneapolis, MN 55455
minette@math.umn.edu

Author: Hoim, Terje, Turnpu, Heino

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Review text:

Let $\lambda = \{\lambda_n\}$, $\mu = \{\mu_n\}$ be two positive sequences increasing to infinity; such sequences are called speeds. A sequence of functions $\{f_n(t)\}$ in the space $M(a, b)$ of measurable functions on an interval $[a, b]$ is said to be λ mes μ -convergent to $f(t)$ on $[a, b]$ if $\lim_{n \rightarrow \infty} \lambda_n \text{mes} \{t : \mu_n |f_n(t) - f(t)| = \alpha\}$ exists for each positive number α ; if $\mu_n = 1$ for each n , then the sequence $\{f_n\}$ is said to be λ mes convergent to $f(t)$. The sequence is said to be mes λ -convergent to f if for each positive number $\alpha \lim_{n \rightarrow \infty} \text{mes} \{t : \lambda_n |f_n(t) - f(t)| \geq \alpha\} = 0$, if $\lambda_n = 1$ for each n the sequence $\{f_n(t)\}$ converges in measure to $f(t)$. The set of mes λ -convergent (λ mes-convergent, λ mes μ -convergent) sequences in $M(a, b)$ is denoted by $c_{mes\lambda}(c_{\lambda mes}, c_{\lambda mes\mu})$; the set of sequences mes λ -convergent, λ mes convergent, λ mes μ convergent to 0) is denoted by $c_{mes\lambda}^o(c_{\lambda mes}^o, c_{\lambda mes\mu}^o)$. The set of λ -convergent sequences is denoted by c^λ . If the sequence $\{f_n\}$ is mes λ -convergent (λ mes-convergent, λ mes μ -convergent, μ mes λ -convergent, λ mes λ -convergent for each speed λ), then $\{f_n\}$ is said to be mes ∞ -convergent (∞ mes-convergent, ∞ mes μ -convergent, μ mes ∞ -convergent, ∞ mes ∞ -convergent). The symbol $c_{mes\infty}(c_{\infty mes}, c_{\lambda mes\infty}, c_{\infty mes\lambda}, c_{\infty mes\infty})$ denotes the set of mes ∞ -convergent (∞ mes-convergent, λ mes ∞ -convergent ∞ mes λ -convergent, ∞ mes ∞ -convergent) sequences in $M(a, b)$. It is shown that the sequence $\{f_n\}$ is in $c_{\lambda mes\infty}^o$ iff $\lim_{n \rightarrow \infty} \lambda_n \text{mes}(\text{support } f_n) = 0$. The sequence $\{f_n\}$ is in $c_{\infty mes\infty}^o$ iff there exists a natural number n_0 such that for $n > n_0$ $f(t)$ vanishes almost everywhere on $[a, b]$.

Let $A = (a_{nk})$ be a summation matrix which transforms a sequence $\{f_n\}$ into a sequence $\{g_n\}$, where $\{g_n\} = A\{f\}_n = \{\sum_{k=0}^{\infty} a_{nk} f_k\}$. The matrix A transforms sequences in $c_{mes\infty}^o$ into sequences in $c_{\mu mes}^o$ iff there exists a natural number M such that $a_{nk} = 0$ for $n > M$, $k > M$. The matrix A transforms sequences in $c_{mes\lambda}$ into sequences in $c_{mes\mu}$ iff it transforms sequences in c^λ into sequences in c^μ and there exists a natural number L such that each row of A has less than L non-zero elements.